# 3 Multiples and factors

## 1 Multiples

The products of a number with the **natural numbers** 1, 2, 3, 4, 5, ... are called the **multiples** of the number.

For example:

$7 \times 1 = 7$

$7 \times 2 = 14$

$7 \times 3 = 21$

$7 \times 4 = 28$

So, the multiples of 7 are: 7, 14, 21, 28, and so on.

**Note:**
The multiples of a number are obtained by multiplying the number by each of the natural numbers.

For example:

- multiples of 2 are 2, 4, 6, 8, …
- multiples of 3 are 3, 6, 9, 12, …
- multiples of 4 are 4, 8, 12, 16, …

*Example 1*

Write down the first ten multiples of 5.
*Solution:*
The first ten multiples of 5 are 5, 10 15, 20, 25, 30, 35, 40, 45, 50.

**Exercise 1**

**a) Write down all the multiples of 6 between 20 and 70**

**b) Write down all the multiples of 7 between 30 and 80**

**c) Write the three smallest multiples of 8 which are over 50**

d) **Write the smallest multiple of 37 which is over 500**

# 2 Factors

A whole number that divides exactly into another whole number is called a **factor** of that number.

For example $20 : 4 = 5$

So, 4 is a factor of 20 as it divides exactly into 20.

We could also consider than $20:5 = 4$

So, 5 is a factor of 20 as it divides exactly into 20.

**Note:**
If a number can be expressed as a product of two whole numbers, then the whole numbers are called **factors** of that number.

For example $20 = 1 \text{x} 20 = 2 \text{x} 10 = 4 \text{x} 5$

So, the factors of 20 are 1, 2, 4, 5, 10 and 20.

*Example 2*

List all the factors of 42.

*Solution:* $42 = 1\text{x}42 = 2\text{x}21 = 3\text{x}14 = 6\text{x}7$

So, the factors of 42 are 1, 2, 3, 6, 7, 14, 21 and 42.

*Example 3*

Is 7 a factor of 15?
*Solution:*

$$15 \div 7 = \begin{cases} \text{quotient } 2 \\ \text{remainder } 1 \end{cases}.$$

Clearly 7 does not divide exactly into 15, so 7 is not a factor of 15

**Exercise 2**
**Write down all the factors of**

a) **60**

b) **20**

c) **100**

# 3 Prime Numbers

If a number has only two different factors, 1 and itself, then the number is said to be a **prime number.**

For example, 7 = 1x7

7 is a prime number since it has only two different factors.

2 = 1x2, 3 = 1x3, 5 = 1x5,… 2,3,5,…. Are prime numbers

**Exercise 3**

**The Sieve of Eratosthenes**

A Greek mathematician, Eratosthenes (276-195 BC), discovered the Sieve which is known as the Sieve of Eratosthenes, it is a method to get prime numbers.

**3.1** We start with a table of whole numbers e.g. from 1 to 200 and cross out the number 1, as it has been done below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ~~1~~ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
| 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 |
| 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |
| 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 |
| 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 |
| 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 |
| 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 |
| 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 |
| 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 |

**3.2.** Circle the number 2 and then cross out all the multiples of 2, as shown below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ~~1~~ | (2) | 3 | ~~4~~ | 5 | ~~6~~ | 7 | ~~8~~ | 9 | ~~10~~ |
| 11 | ~~12~~ | 13 | ~~14~~ | 15 | ~~16~~ | 17 | ~~18~~ | 19 | ~~20~~ |
| 21 | ~~22~~ | 23 | ~~24~~ | 25 | ~~26~~ | 27 | ~~28~~ | 29 | ~~30~~ |
| . . . . . | | | | | | | | | |
| . . . . . | | | | | | | | | |
| 181 | ~~182~~ | 183 | ~~184~~ | 185 | ~~186~~ | 187 | ~~188~~ | 189 | ~~190~~ |
| 191 | ~~192~~ | 193 | ~~194~~ | 195 | ~~196~~ | 197 | ~~198~~ | 199 | ~~200~~ |

**3.3.** The next number that is not crossed out is 3. Circle it and then cross out all the multiples of 3: 3, 6, 9, 12....

**3.4.** The next number that is not crossed out is 5. Circle it and then cross out all the multiples of 5: 5, 10, 15, 20….

**3.5.** The next number that is not crossed out is 7. Circle it and then cross out all the multiples of 7.

**3.6**. Continue this process until there is no number to be crossed.

**3.7**. Make a list of all the circled numbers.

**3.8.** Write the factors of each of the circled numbers.

**3.9.** Make a list of the first twenty crossed out numbers and write the factors of these numbers.

**3.10** What do you observe about the number of factors of the circled numbers and the crossed out numbers? Write a brief sentence in your own words.

**3.11**. What name is given to the circled numbers?

**3.12**. What name is given to the crossed out numbers?

**3.13.** How many prime numbers are less than 100?

## 4 Tests of divisibility

| One number is divisible by: |
| --- |
| **2** If the last digit is 0 or is divisible by 2, (0, 2, 4, 8). |
| **3** If the sum of the digits is divisible by 3. |
| **4** If the last two digits are divisible by 4. |
| **5** If the last digit is 0 or is divisible by 5, (0,5). |
| **9** If the sum of the digits is divisible by 9. |

**8** If the half of it is divisible by 4.

**6** If it is divisible by 2 and 3.

**11** If the sum of the digits in the even position minus the sum of the digits in the uneven position is 0 or divisible by 11.

**Exercise 4** Find which of the numbers: 239 300 675 570 800 495

888 6402 2088 are multiples of:

a) 3 →

b) 2 →

c) 5 →

d) 4 →

e) 11 →

f) 9 →

**Exercise 5** Find the factor decomposition of the following numbers:

| 24 | 123 | 420 | 4752 |
|---|---|---|---|
| | | | |

24 = 123 = 420 = 4752 =

## 5 Common Multiples

Multiples that are common to two or more numbers are said to be **common multiples**.

E.g. Multiples of 2 are 2, 4, **6**, 8, 10, **12**, 14, 16, **18**, …
Multiples of 3 are 3, **6**, 9, **12**, 15, **18**, …

So, common multiples of 2 and 3 are 6, 12, 18, …

*Example 4*

Find the common multiples of 4 and 6.
*Solution:*
Multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, **36**, …
Multiples of 6 are 6, **12**, 18, **24**, 30, **36**, …
So, the common multiples of 4 and 6 are 12, 24, 36, …

**Exercise 6**

**a) Find the sequence of the common multiples of 3 and 5.**

**b) Find the sequence of the common multiples of 12 and 9.**

## Lowest common multiple

The smallest common multiple of two or more numbers is called the **lowest common multiple** (LCM).

E.g. Multiples of 8 are 8, 16, **24**, 32, …
Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, **24**, …

LCM of 3 and 8 is 24

## Method I (for small numbers)

To find the lowest common multiple (LCM) of two or more numbers, list the multiples of the largest number and stop when you find a multiple of the other number. This is the LCM.

*Example 5*

Find the lowest common multiple of 6 and 9.
*Solution:*
List the multiples of 9 and stop when you find a multiple of 6.

Multiples of 9 are 9, **18**, …
Multiples of 6 are 6, 12, **18**, …
LCM of 6 and 9 is 18

*Example 6*

Find the lowest common multiple of 5, 6 and 8.
*Solution:*
List the multiples of 8 and stop when you find a multiple of both 5 and 6.
Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120, …
Stop at 120 as it is a multiple of both 5 and 6.
So, the LCM of 5, 6 and 8 is 120.

**Exercise 7 Find the LCM of**

**a) 6 and 8**

**b) 10 and 20**

**c) 8 and 12**

## Method II (General)

To find the lowest common multiple (LCM) of higher numbers:

- Find the prime factor decomposition.

- Choose the **non common factors** and the **common factors with the highest exponents.**

*Example 7*

Find the lowest common multiple of 18 and 24.
*Solution:*

$18 = 2 \cdot 3^2$
$24 = 2^3 \cdot 3$ So, the LCM of 18 and 24 is $LCM = 2^3 \cdot 3^2 = 72$.

**Exercise 8 Find the LCM of**

**a) 150 and 350**

**b) 100 and 120**

**c) 120, 480 and 180**

# 6 Common Factors

Factors that are common to two or more numbers are said to be **common factors**.

For example $4 = 1x4 = 2x2$
$6 = 1x6 = 2x3$

- Factors of 4 are 1,2 and 4
- Factors of 6 are 1, 2, 3 and 6

So, the common factors of 4 and 6 are 1 and 2

*Example 8*

Find the common factors of 10 and 30.
*Solution:*
$10 = 1x10 = 2x5$
$30 = 1x30 = 2x15 = 5x6 = 3x10$

So, the common factors of 10 and 30 are **1, 2, 5** and **10**.

*Example 9*

Find the common factors of 26 and 39.
*Solution:*
$26 = 1x26 = 2x13$
$39 = 1x39 = 3x13$
So, the common factors of 26 and 39 are **1** and **13**.

# 7 Highest Common Factor

The largest common factor of two or more numbers is called the **highest common factor** (HCF).

For example $8 = 1x8 = 2x4$
$12 = 1x12 = 2x6 = 3x4$

- Factors of 8 are 1,2, 4 and 8
- Factors of 12 are 1, 2, 3, 4, 6 and 12

So, the common factors of 8 and 12 are 1, 2 and 4 HCF is **4**

*Example 10*

Find the highest common factor of 14 and 28.
*Solution:*
$14 = 1x14 = 2x7$
$28 = 1x28 = 2x14 = 4x7$
HCF = **14**

To find the **Highest Common Factor** of higher numbers:
- Find the prime factor decomposition.
- Choose only the **common factors with the lowest exponents.**

**Exercise 9 Find the HCF and the LCM of:**

**a) 18 and 24**

**b) 180 and 40**

**c) 60, 320 and 140**

## EXTRA EXERCISES

**1. Which numbers between 37 and 74 have a factor of 3?**

**2. Which of these is a multiple of 6?**

**122, 28, 30, 402, 634, 348, 10,500**

**3. List all the prime numbers between 20 and 50.**

**4. List all the factors of:**

**a) 12 b) 30 c) 66 d) 200**

**5. In a bus station there is a bus leaving for London every 45 minutes and one leaving for Brighton every 60 minutes. If a bus to London and a bus to Brighton leave at the same time, how many minutes will it be before two buses leave again at the same time?.**

**6. Find the prime factorization of both 156 and 250. What is the HCF of these numbers? What is the LCM?**

**7. List all the common factors of 30 and 75. What is the HCF of 30 and 75? What is the LCM of 30 and 75?**

**8. Find the HCF and the LCM of 36 and 90.**

**9. Lara has a day off every six days and Dave has a day off every eight days, if they both have a day off on the first of November, which day will they have the same day off again?**

**10. What numbers that are less than 100 are multiples of 3 and 5?**

**11. How many different rectangles with an area of 36 $cm^2$ using only whole numbers (centimetres), can be made?**

**12. Three traffic lights are placed along the same avenue at three different crosses. The first one changes every 20 seconds, the second, every 30 seconds and the third every 28 seconds. They have changed to green simultaneously. How long does it take until they change again at the same time? Explain your answer.**

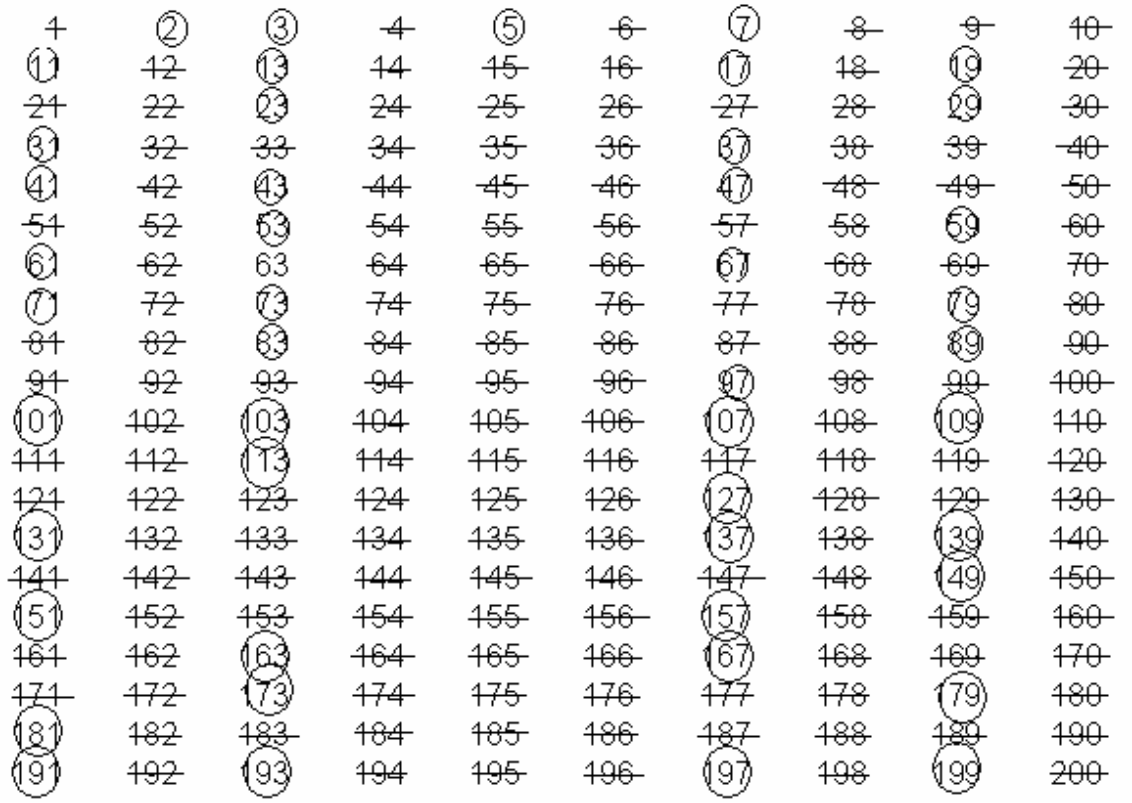
**13. Marta has 12 red, 30 green and 42 yellow marbles and she wants to put them in boxes, as many as possible, all the boxes with the same amount of each colour and with no marbles remaining. How many boxes will she have? How many marbles of each colour are there in each box?**

## Solutions

**Exercise 1** a) 24, 30, 36, 42, 48, 54, 60 and 66; b) 35, 42, 49, 56, 63, 70 and 77 c) 56, 64 and 72; d) 518

**Exercise 2** a) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60; b) 1, 2, 4, 5, 10 and 20; c) 1, 2, 4, 5, 10, 20, 25, 50 and 100

**Exercise 3: 3.1 to 3.7**



**3.8.** Only the number multiplied by 1

**3.9.** 1=1, 4=2x2, 6=3x2, 8=2x2x2, 9=3x3, 10=2x5, 12=2x2x3, 14=2x7, 15=3x5, 16=2x2x2x2, 18=3x3x2, 20=2x2x5, 21=3x7, 22=2x11, 24=2x2x2x3, 25=5x5, 26=2x13, 27=3x3x3, 28=2x2x7, 30=2x3x5
**3.11**. Prime numbers **3.12**. Composite numbers **3.13.** 25 numbers

**Exercise 4** Multiples of: a) 3 → 300, 675, 570, 495, 888, 6402, 2088
b) 2 → 300, 570, 800, 888, 6402, 2088; c) 5 → 300, 675, 570, 800, 495
d) 4 → 300, 800, 888, 2088; e) 11 → 495, 6402; f) 9 → 675, 495, 2088

**Exercise 5** $24 = 2^3 \cdot 3$; $123 = 3 \cdot 41$; $420 = 2^2 \cdot 3 \cdot 5 \cdot 7$; $4752 = 2^4 \cdot 3^3 \cdot 11$

**Exercise 6** a) 15, 30, 45, …; b) 36, 72, 108, …

**Exercise 7** a) 24; b) 20; c) 24.
**Exercise 8** a) 1050; b) 600; c) 1440

**Exercise 9** a) 6 and 72; b) 20 and 360; c) 20 and 6720.

## EXTRA EXERCISES

**1.** 39, 42, 45, 48, 51, 54, 57, 60, 63, 66 and 69

**2.** 30, 402, 348 and 10,500

**3.** 23, 29, 31, 37, 41, 43 and 47.

**4.** a) 1, 2, 3, 4, 6 and 12. b) 1, 2, 3, 5, 6, 10, 15 and 30. c) 1, 2, 3, 6, 11, 22, 33 and 66. d) 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100 and 200.

**5.** LCM(45,60) = 180 min.

**6.** $156 = 2^2 \cdot 3 \cdot 13$, $250 = 2 \cdot 5^3$, $\text{HCF}(156,250) = 2$, $\text{LCM}(156,250) = 19500$

**7.** 1, 3, 5 and 15**,** $\text{HCF}(30,75) = 15$, $\text{LCM}(30,75) = 150$.

**8.** $\text{HCF}(36,90) = 18$, $\text{LCM}(36,90) = 180$.

**9.** $\text{LCM}(6,8) = 24$, they both will have a day off again on the $25^{\text{th}}$ of november.

**10.** 15, 30, 45, 60, 75 and 90.

**11.** 1x36, 2x18, 3x12, 4x9 and 6x6 cm.

**12.** LCM(20,30,28) = 420 secons or 7 minutes.

**13.** $\text{HCF}(12,30,42) = 6$, so 6 boxes, each one containing 2 red, 5 green and 7 yellow marbles.